Random Monotone Cellular Automata

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One of the simplest cellular automata is *bootstrap percolation with infection parameter* r, introduced by Chalupa, Leith and Reich in 1979. This process is an oversimplified model of the spread of an infection on a graph: each site may be *healthy* or *infected*, with an infected site remaining infected for ever, and a healthy site getting infected if it has at least r infected neighbours. At the start of the process, infected sites are chosen at random, with the same probability p. We say that this process *percolates* if each site becomes infected after some time.

Bootstrap percolation has been studied extensively on sequences (G_n) of finite grid graphs, with the main interest in the *critical probability* $p_c(n)$, below which G_n is very unlikely to percolate, and above which it is very likely to do so.

Recently, with Smith and Uzzell, I initiated the study of a far-reaching generalization of bootstrap percolation on lattices and lattice-like finite graphs. The only assumptions we made about such a process is that it is local, homogeneous and *monotone*. Surprisingly, much can be proved about these very general processes; in particular, they can be classified into three classes, telling us much about the critical probability. In two dimensions such a classification was given by Smith, Uzzell, Balister, Przykucki and me. Even more recently, Duminil-Copin, Morris, Smith and I have gone much further: we have proved fairly precise results about the process in all dimensions, indicating that a rich theory of 'universality theory' is waiting to be discovered.

In my talk I shall start with a brief review the basic results on bootstrap percolation, and then I shall give a brief description of the phenomena concerning monotone, local, and homogeneous cellular automata on the two-dimensional torus and beyond.